# Optimal Driving Speed with the Chevy Bolt

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August 19, 2022

## 1 Statement of the Problem

Fellow forum member @jefro asked the following question[1]

```
Can I drive really fast to a station and make up for
    the extra power lost by using faster charging than
    my increased consumption used over time? I'd guess
    the super fast DC cars could make up time but my
    calculations show net zero.
```

To try to answer this question, we aim to determine the vehicle speed that minimizes travel time and charge time given distance and initial state of charge (SOC).

## 2 Estimating Charge Time

Electric vehicles (EVs) that charge using Direct Current Fast Charging (DCFC) limit the charge rate according to a charge curve. This is presumably done to prevent damage to the battery. The charge curve $C(s)$ associates SOC $0 \leq s \leq 1$ to a charge rate (kW). The Chevy Bolt can draw up to a maximum of 55 kW while charging using DCFC and follows the charge curve shown in figure 1. As the battery approaches full charge, the charge rate slows significantly.

A simple way to simulate charging is to follow the charge curve using small intervals of time $h$ (hours). For a given SOC $s$, you draw $C(s) \times h$ kWh from the DCFC station over $h$ hours. If your car has a battery capacity of $K$ kWh, that translates to $\frac{C(s)h}{K}$ SOC gained. Your new SOC is then $s + \frac{C(s)h}{K}$. You can repeat this process until you reach a target SOC $s_{\text{final}}$. Keeping track of how many $h$ intervals you charged gives a total of $Nh$ hours where $N$ is the number of times you repeated.

If $s_0$ is your initial SOC and you wish to charge to SOC $s_{\text{final}}$, this process can be concisely given as the following recurrence

$$\left.\begin{aligned} t_{n+1} &= t_n + h \\ s_{n+1} &= s_n + \tfrac{C(s_n)h}{K} \end{aligned}\right\} \forall n = 0, 1, 2, \ldots, N-1,\ s_n < s_{\text{final}}$$

[1]https://www.chevybolt.org/threads/dcfc-charging-is-an-ode.44954/post-743750

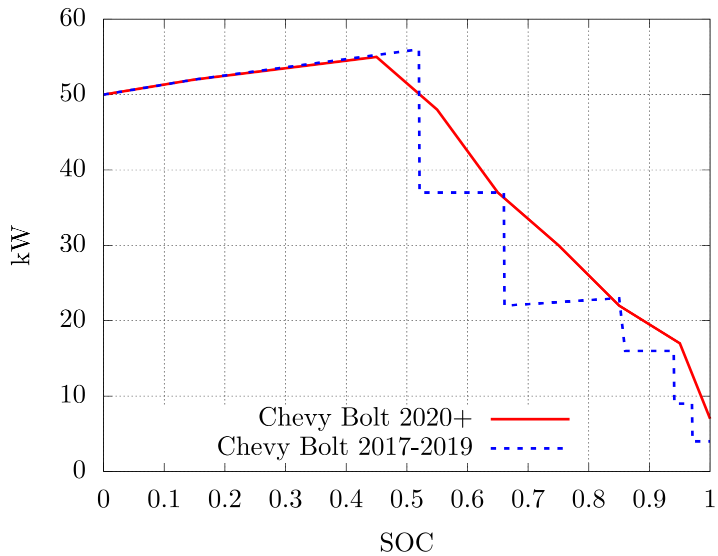



Figure 1: These are the Chevy Bolt charge curves. Power drawn from DCFC varies by SOC. Batteries replaced under recall likely use the 2020+ charge curve. These curves originate from https://allev.info/wp-content/uploads/2020/03/DCFC2020Curve-1024x576.png

where $t_N$ is the total amount of time (hours) used to charge to SOC $s_N \approx s_{\text{final}}$. With a bit of rewriting, this recurrence becomes a differential equation

$$\begin{aligned} \frac{s_{n+1} - s_n}{h} &= \frac{s(t_n + h) - s(t_n)}{h} \\ &= \frac{C(s(t_n))}{K} \end{aligned}$$

Letting $h \to 0$ and replacing $t_n$ with arbitrary $t$ gives the ordinary differential equation (ODE)

$$s'(t) = \frac{C(s(t))}{K} \tag{1}$$

The recurrence to simulate DCFC charging is actually the Euler method used to solve general ODEs.

Another way to estimate charging time from (1) is to solve for time as a function of SOC $t(s)$ instead of $s(t)$. With a bit of rewriting we have

$$\frac{dt}{ds} = \frac{K}{C(s)}$$

Integrating this expression gives us total time to charge from 0 SOC to $s$ SOC

$$t(s) = \int_0^s \frac{K}{C(u)} du \tag{2}$$

The time needed to charge from $s_0$ to $s_{\text{final}}$ is simply the difference $t(s_{\text{final}})-t(s_0)$ (for $s_{\text{final}} \geq s_0$).

## 3 Optimal Speed for a Round Trip

We wish to determine optimal driving speed $v$ (mph) to drive distance $d$ (miles) and back so that travel time and charging time is minimized. And we wish to charge just enough for the return trip. To solve this, we need to know how much power is drained (kW) to maintain speed $v$. Denote $D(v)$ to be the power drain curve. This curve is given for the Chevy Bolt[2] and seems to roughly correspond to the following quadratic function

$$D(v) = 0.0071429v^2 - 0.25v + 4.2857143 \quad 25 \leq v \leq 80$$

which was determined by fitting $D(25) = 2.5,\ D(45) = 7.5,\ D(80) = 30$.

The time needed to travel distance $d$ with speed $v$ is given as

$$t_{\text{travel}} = \frac{d}{v}$$

The power consumed traveling with constant speed $v$ for $t_{\text{travel}}$ hours is then $D(v)t_{\text{travel}}$ kWh. This corresponds to a loss of $\frac{D(v)t_{\text{travel}}}{K}$ SOC. To reduce charging time, we only need enough SOC that will enable a return trip of $d$ distance at speed $v$ so that we return with no less than some small margin SOC $s_{\text{margin}}$ (e.g. $s_{\text{margin}} = 0.15$). This gives the destination SOC $s_{\text{dest}}$ and target charge SOC $s_{\text{final}}$ as

$$s_{\text{dest}} = s_0 - \frac{D(v)t_{\text{travel}}}{K} \tag{3}$$

$$s_{\text{final}} = s_{\text{margin}} + \frac{D(v)t_{\text{travel}}}{K} \tag{4}$$

where $s_0$ is the initial SOC.

We wish to minimize the following

$$v^* = \min_{25 \leq v \leq 80} \ell(v) \qquad \ell(v) = 2t_{\text{travel}} + \max\{0, t(s_{\text{final}}) - t(s_{\text{dest}})\} + I(t(s_{\text{final}}) > t(s_{\text{dest}}))t_{\text{overhead}} \tag{5}$$

where $I(\cdot)$ is the indicator function (1 if true, 0 if false), $t_{\text{overhead}}$ is the time spent getting the DCFC station working if charging is needed, $2t_{\text{travel}}$ describes the round trip time and $t(s)$ is defined in equation (2). The $\max\{0, t(s_{\text{final}})-t(s_{\text{dest}})\}$ term prevents nonsensical (negative) charging time. Since $\ell(v)$ is piecewise differentiable, we can turn (5) into a root-finding problem by solving for critical

[2] https://www.chevybolt.org/threads/speed-vs-energy-efficiency.5866/post-55986

points $\ell'(v) = 0$. The derivative is given as

$$\ell'(v) = -\frac{2d}{v^2} + I(t(s_{\text{final}}) > t(s_{\text{dest}})) \left( t'(s_{\text{final}}) \frac{D'(v)\frac{d}{v} - D(v)\frac{d}{v^2}}{K} - t'(s_{\text{dest}}) \frac{-D'(v)\frac{d}{v} + D(v)\frac{d}{v^2}}{K} \right) \tag{6}$$

where $I(\cdot)$ is the indicator function. The derivative $t'(s) = \frac{K}{C(s)}$ which follows from (2).

# 4 Optimal Speed for a Multi-Stop Trip

The round trip scenario can be generalized to solve a multi-stop trip spanning arbitrarily large distances (i.e. thousands of miles). Denote $d_0, d_1, \ldots, d_M$ to be the distances between $M$ consecutive stops. Denote $P_0, P_1, \ldots, P_{M-1}$ to be first, second, ..., and M$^{th}$ stop respectively ($P$ for point). A visual depiction of the problem can be seen in figure 2. In the round trip scenario our problem parameters would be $d_0 = d_1$ distances with $M = 1$ stop $P_0$. We wish to choose an optimal speed so that total travel and charge times are minimal. Much like the problem setup in section 3, we have an initial SOC $s_0$ when traveling first commences (i.e. from home). For any given stop $P_m$ we need to determine minimum charging SOC $s_{m+1}$ to travel $d_{m+1}$ distance at $v$ speed to arrive with no less than $s_{\text{margin}}$ SOC. Thus for each stop $P_m$ we need to charge to an SOC of

$$s_{m+1} = s_{\text{margin}} + \frac{D(v)t_{m+1}}{K} \quad \forall m = 0, 1, \ldots, M-1$$

where $t_{m+1} = \frac{d_{m+1}}{v}$ is the time needed to travel $d_{m+1}$ distance at speed $v$. The arrival SOC $\hat{s}_m$ at stop $P_m$ is

$$\hat{s}_m = s_m - \frac{D(v)t_m}{K} \quad \forall m = 0, 1, 2, \ldots, M$$

Owing to the charging strategy, you should normally arrive at stop $P_m,\ m > 0$ with around $\hat{s}_m = s_{\text{margin}}$ SOC with the first stop $P_0$ being the exception. However, this is not true when $d_m$ are trivial distances (e.g. 10 miles). When this happens, many of the initial stops $P_m$ will not require any charging and it would be as if the charging stop did not exist. To be completely general, let us entertain this silly possibility in the solution to our optimization. To deal with this possibility, we modify charging SOC $s_{m+1}$ as follows

$$s_{m+1} = \max\left\{ s_{\text{margin}} + \frac{D(v)t_{m+1}}{K}, \hat{s}_m \right\} \tag{7}$$

Thus the target charge SOC $s_{m+1}$ will be the arrival SOC for such trivial *stops*.

Now we can pose the general optimization problem similar to (5)

$$v^* = \min_{25 \le v \le 80} \ell(v) \tag{8}$$

$$\ell(v) = \sum_{m=0}^{M} t_m + \sum_{m=0}^{M-1} \left( t(s_{m+1}) - t(\hat{s}_m) + I(t(s_{m+1}) > t(\hat{s}_m)) t_{\text{overhead}} \right) \tag{9}$$

where $t_{\text{overhead}}$ is the time spent getting the DCFC station working if charging is needed. Because we modify $s_{m+1}$ in (7), charge time can never be negative at any stop $P_m$ and thus no $\max\{\cdot\}$ trickery is needed. This function is piecewise differentiable and the derivative is

$$\ell'(v) = -\sum_{m=0}^{M} \frac{d_m}{v^2} + \sum_{m=0}^{M-1} \left( t'(s_{m+1}) s'_{m+1} - t'(\hat{s}_m) \hat{s}'_m \right) \tag{10}$$

where $s'_{m+1}$, $\hat{s}'_m$ are given as

$$s'_{m+1} = \begin{cases} \frac{D'(v)\frac{d_{m+1}}{v} - D(v)\frac{d_{m+1}}{v^2}}{K} & s_{m+1} > \hat{s}_m \\ s'_m - \frac{D'(v)\frac{d_m}{v} - D(v)\frac{d_m}{v^2}}{K} & \text{otherwise} \end{cases}$$

$$\hat{s}'_m = s'_m - \frac{D'(v)\frac{d_m}{v} - D(v)\frac{d_m}{v^2}}{K}$$

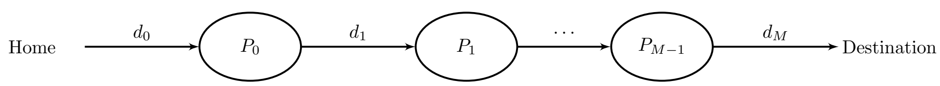



Figure 2: This figure shows the problem setup for the multi-stop problem.

# 5 Methods

To respect the minimization bounds $25 \le v \le 80$, the bisection method was used to find roots of $\ell'(v)$. The integral given in (2) is estimated using the trapezoid rule (although the Euler method works too). Linear interpolation is used to estimate function values where data points are absent (e.g. $C(s)$).

# 6 Results

The optimal speed and timings for the *round trip problem* (see section 3) are shown in figure 3. Following this are the expected destination and target charge

SOCs shown in figure 4. Lastly, time differences between optimal speed and fixed 60 mph and 80 mph speeds are shown in figure 5.

With respect to the *multi stop problem* (see section 4), the optimal speed and timings are shown in figure 6 for varying charge stop distances. The target charge SOCs and per-stop charge times are shown in figure 7 and lastly the time differences between optimal speed and 60 mph and 80 mph speeds are shown in figure 8.

The maximum relative total time differences between driving the optimal speed and a fixed speed (60 mph or 80 mph) for both problems are shown in table 1.

| | 60 mph | 80 mph |
|---|---|---|
| Round Trip | 33.3% | 2.2% |
| Multi Stop | 8.2% | 9.8% |

Table 1: Maximum relative total time difference between optimal speed compared to driving a fixed speed for the round-trip and multi-stop problems. These indicate relatively how much more total time is needed to drive a fixed speed compared to driving the optimal speed (at worst).

## 7 Discussion

Because the Chevy Bolt has such a slow DCFC charge rate, it is easy to suppose that the charge time should dominate total trip time when driving faster (e.g. 80 mph). And there is an optimal driving speed that reduces the time spent charging (see figures 3,6). However, figures 5,8 show that travel time and charging time cancel each other out when driving fast. The time saved driving fast is approximately consumed by charging time in comparison to driving the optimal speed ($< 10\%$ slower). Remarkably, this is regardless of needing to charge to near 100% SOC as is hinted in figures 4,7. For example, if you needed to charge to $\approx 85\%$ for a 140 mile round trip driving an optimal $\approx 71$ mph, you would need to charge to $\approx 95\%$ SOC driving 80 mph and you would only be 2% slower in total time at worst. With respect to the multi-stop problem, even driving a slower 60 mph does not seem to make a big difference in total time compared to driving the optimal speed ($< 10\%$ slower). Faster driving, however, does drain more power and limit distances between charging stops. For example, the plot in figure 4 stops at 140 miles because driving 80 mph will empty the battery beyond that (when starting with 80% SOC).

So you have a choice, do you want to spend more time charging or more time driving? Driving faster will lead to reduced driving time and more charging time while driving slower will lead to more driving time and less charging time. The effect of driving speed is negligible and you will be no worse than 10% slower than optimal (if you drive between 60 and 80 mph anyway).

## 8 Limitations

These simulations assume driving on flat terrain at a precise constant speed in good weather with no wind. The power drain and charge curves are unofficial and may be inaccurate. Inaccuracies in these curves can drastically change the simulation results. Simulations also assume a constant 10 minutes overhead at charging stops. Some charging stops may actually require calls to customer service and consume more than 10 minutes while others may work flawlessly and consume less than 10 minutes.

## 9 Disclaimer

These are simulations and you should be especially careful when trying to validate my results in the real world. And I do not advocate breaking traffic laws for the sake of optimality. Please see the license for this work in `LICENSE.txt`.

## 10 Conclusion

While there is an optimal speed to drive simple round trips or multi-stop trips, the overall time advantages are quite small ($< 10\%$) compared to just driving fast (80 mph) or conservatively (60 mph) in the case of the multi-stop scenario.

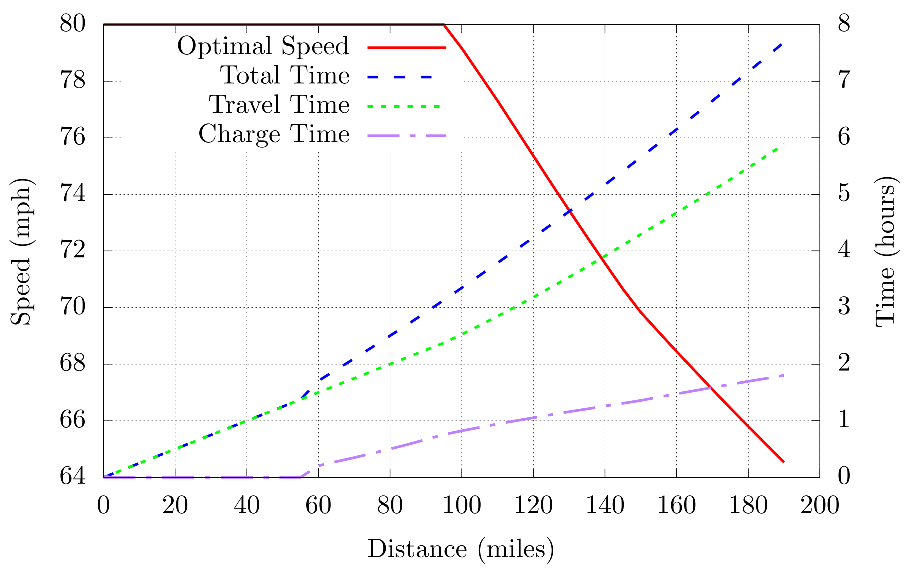



Figure 3: This plot shows the optimal speed to drive a 2020 Chevy Bolt so that total time (travel time + charge time) is minimized when driving a given distance and back. Upon reaching its destination, the Bolt is minimally charged for a return trip so that it arrives with no less than 15% SOC ($s_{\text{margin}} = 0.15$). The Bolt is assumed to start with an initial 80% SOC ($s_0 = 0.8$). The charging stop is assumed to consume 10 additional minutes if charging is needed.

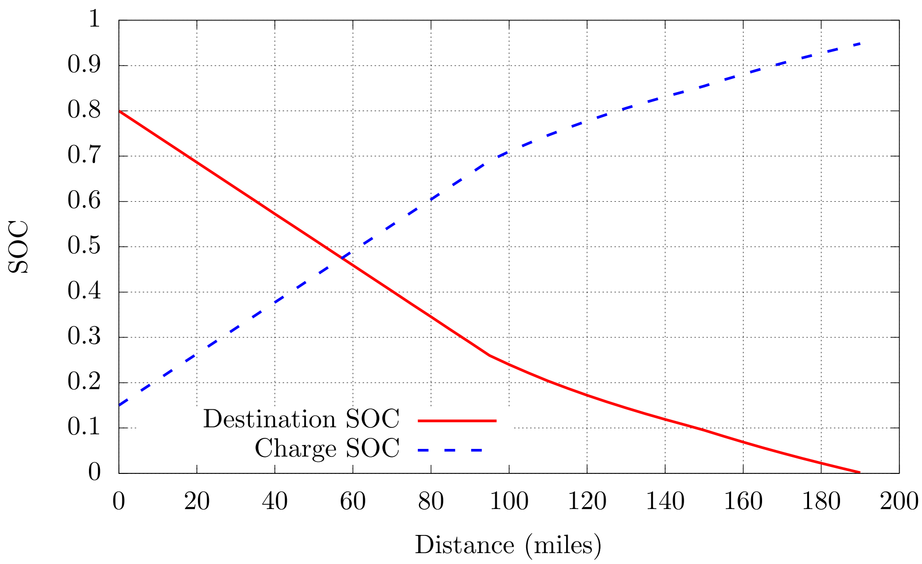
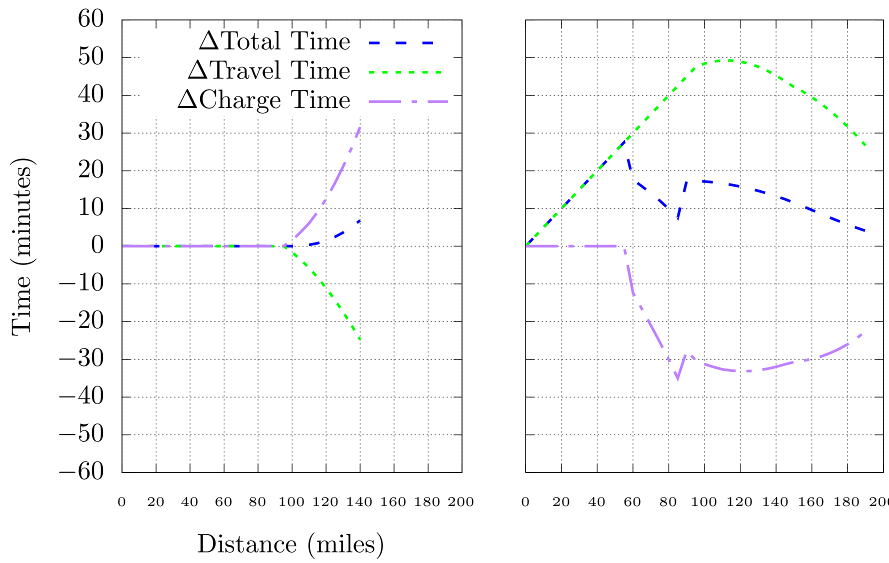



Figure 4: This plot complements figure 3 and shows the destination and charge-to SOC when driving a 2020 Chevy Bolt a given distance and back at a time-optimal speed.




(a) 80 mph

(b) 60 mph

Figure 5: This is a plot of the total, travel and charge time differences driving a 2020 Chevy Bolt a given distance and back at the optimal speed and a fixed speed of (a) 80 mph and (b) 60 mph. The initial SOC is assumed to be 80% ($s_0 = 0.8$) and the Bolt is minimally charged for a return trip so that it arrives with no less than 15% SOC ($s_{\mathrm{margin}} = 0.15$). The lines extend until destination SOC would be 0%. For example, there is not enough battery capacity to drive 200 miles at 80 mph. Positive time differences mean slower than optimal.

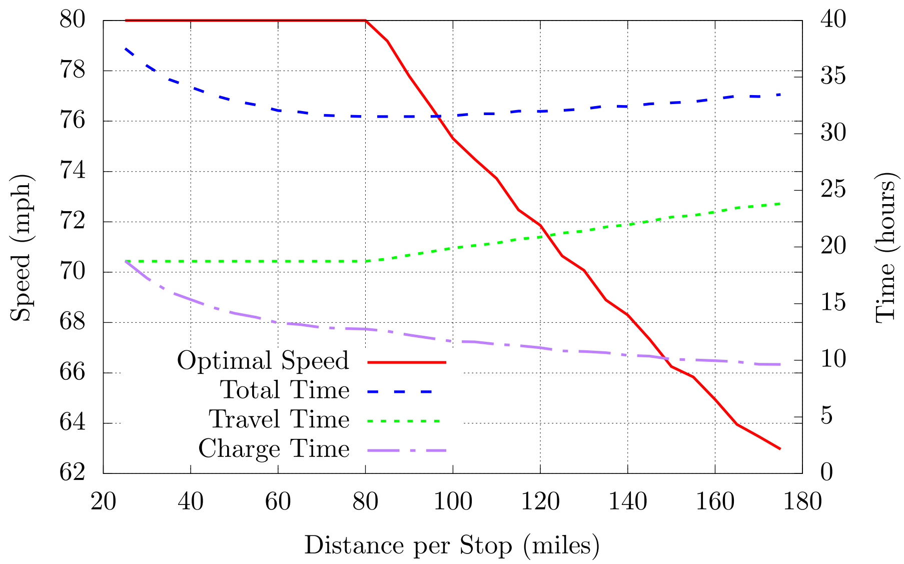



Figure 6: This plot shows the optimal speed to drive a 2020 Chevy Bolt so that total time (travel time + charge time) is minimized when driving 1500 miles with periodic evenly-spaced stops. Upon reaching each stop, the Bolt is minimally charged to reach the next stop so that it arives with no less than 15% SOC ($s_{\text{margin}} = 0.15$). The Bolt is assumed to start with an initial 100% SOC ($s_0 = 1.0$). Each stop is assumed to consume 10 additional minutes if charging is needed.

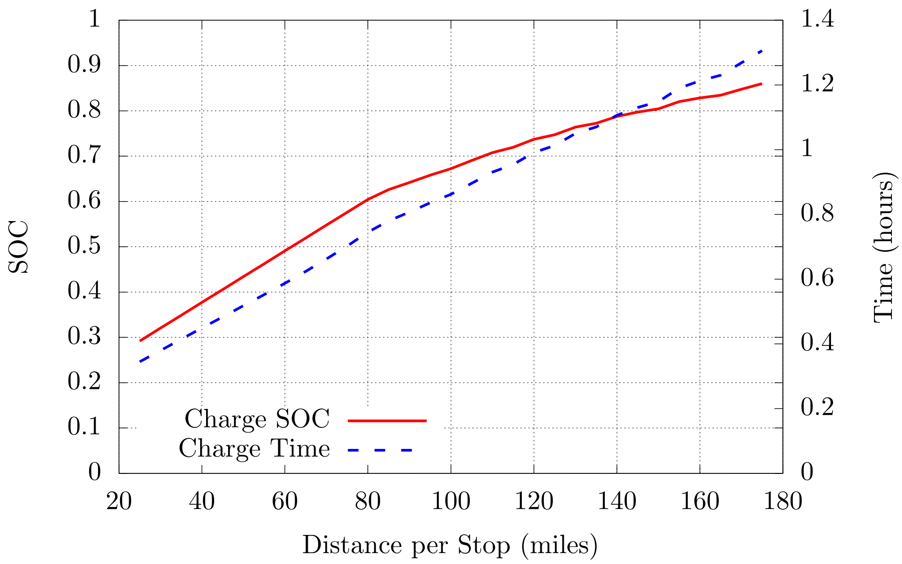
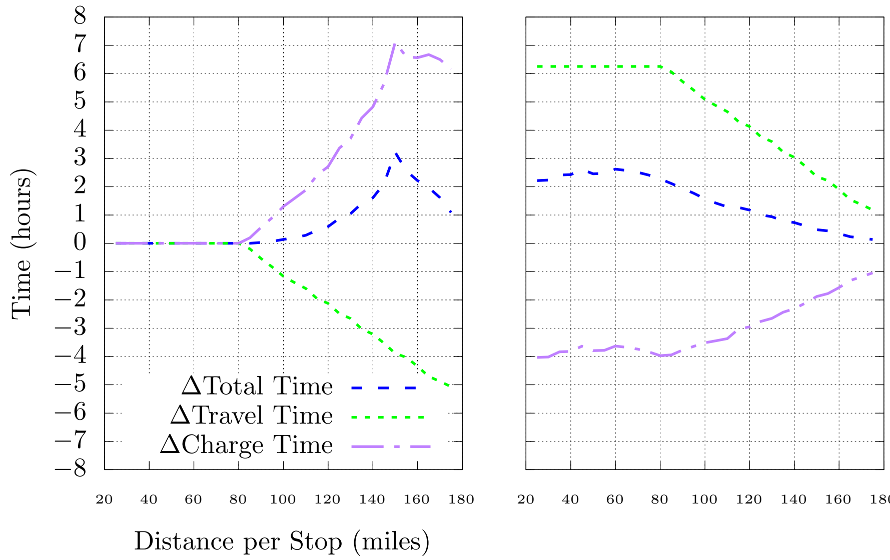



Figure 7: This plot complements figure 6 and shows the charge-to SOC and charge time per-stop when driving a 2020 Chevy Bolt a given distance per stop at a time-optimal speed.




(a) 80 mph

(b) 60 mph

Figure 8: This is a plot of the total, travel and charge time differences driving a 2020 Chevy Bolt 1500 miles and charging at a given distance between stops when driving at the optimal speed and a fixed speed of (a) 80 mph and (b) 60 mph. The initial SOC is assumed to be 100% ($s_0 = 1.0$) and the Bolt is minimally charged to reach the next stop so that it arrives with no less than 15% SOC ($s_{\text{margin}} = 0.15$). When driving 80 mph, the total time difference drops at larger distances because it is no longer possible to respect the 15% margin rule (e.g. you cannot charge to 115%). Positive time differences mean slower than optimal.